NUMERICAL METHODS

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SOLUTION OF LINEAR EQUATION

LINEAR EQUATION

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A linear equation in n unknowns is an equation of the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$$

where a1, a2,....an and b are real (or complex) constants and x1, x2,....xn are unknowns.

Ex: $2x + 3y = 5$ **Linear equation in 2 variables**

$x - y + z = 4$ **Linear equation in 3 variables**

A system of linear equations consists of several such equations taken together

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

MATRIX REPRESENTATION

In compact form,

$$AX = B$$

where

A = coefficient matrix (m×n)

X = $[x_1, x_2, \ldots, x_n]^T$ = vector of unknowns

B = $[b_1, b_2, \ldots, b_m]^T$ = vector of constants

SOLUTION OF LINEAR EQUATION

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- **Gauss's Elimination Method**
- **Gauss's Seidal Iterative Method**

GAUSS'S ELIMINATION METHOD

Gauss's Elimination Method is a direct method of solving a system of linear equations.

It works by reducing the system to an upper triangular form using forward elimination, and then solving the equations using back substitution.

> Gauss elimination is named after German mathematician and scientist Carl Friedrich Gauss.

STEPS OF GAUSS'S ELIMINATION METHOD

- **Form the Augmented Matrix [A | B].**
- **Forward Elimination**
  - Choose a pivot (diagonal element).
  - Make all elements below pivot = 0 by row operations.
  - Repeat for each column until upper triangular form is obtained.
- **Back Substitution**
  - Start from last equation (with one variable).
  - Substitute upward to find all variables.

EXAMPLE OF GAUSS'S ELIMINATION METHOD

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The system of equations are

$$2x + y - z = 8$$

$$-3x - y + 2z = -11$$

$$-2x + y + 2z = -3$$

Then Augmented Matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right]$$

Now, Forward Elimination

Make pivot = 2 (first element)
Eliminate below it

$$R2 \to R2 + \tfrac{3}{2}R1, \quad R3 \to R3 + R1$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 2 & 1 & 5 \end{array}\right]$$

Next, pivot = 0.5 (row 2)
Eliminate below it

$$R3 \to R3 - 4R2$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

Now we have upper triangular form

Now, Back Substitution

From last row,
-z=1 ⇒ z=-1

From second row
0.5y+0.5z=1
0.5y-0.5=1 ⇒ y=3

From first row
2x+y-z=8
2x+3-(-1)=8 ⇒ 2x+4=8 ⇒ x=2

And the final answer

x=2, y=3, z=-1

GAUSS'S SEIDAL ITERATIVE METHOD

Gauss Seidel Method is an iterative method used to solve a system of linear equations.

Instead of solving directly (like Gauss elimination), it starts with an initial guess and improves the solution step by step until the values converge (become stable).

Gauss Seidel is named after the German mathematicians Carl Friedrich Gauss and Philipp Ludwig von Seidel.

STEPS OF GAUSS'S SEIDAL ITERATIVE METHOD

- **Rearrange Equations**
  - Write each equation so that one variable is expressed in terms of the others.
- **Initial Guess**
  - Assume starting values for all variables (usually 0, 0, 0...).
- **Iterative Update**
  - Compute x1 using the latest values of other variables.
  - Compute x2 using the new x1.
  - Compute x3 using the new x1,x2.
  - Continue for all variables.
- **Repeat Iterations**
  - Perform steps until the values stop changing (difference is very small).
- **Final Solution**
  - The stable (converged) values are taken as the solution of the system.

EXAMPLE OF GAUSS'S SEIDAL ITERATIVE METHOD

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The system of equations are

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$

Then Rearrange Equations

$$x = \frac{1}{10}(12 - y - z)$$

$$y = \frac{1}{10}(13 - 2x - z)$$

$$z = \frac{1}{10}(14 - 2x - 2y)$$

Now, Initial Guess

Take (x0,y0,z0)=(0,0,0)

Iteration 1

$$x_1 = \frac{1}{10}(12 - 0 - 0) = 1.2$$

$$y_1 = \frac{1}{10}(13 - 2(1.2) - 0) = \frac{13 - 2.4}{10} = 1.06$$

$$z_1 = \frac{1}{10}(14 - 2(1.2) - 2(1.06)) = \frac{14 - 2.4 - 2.12}{10} = 0.948$$

So, after 1st iteration,
(x,y,z)=(1.2, 1.06, 0.948)

Iteration 2

$$x_2 = \frac{1}{10}(12 - 1.06 - 0.948) = 0.999$$

$$y_2 = \frac{1}{10}(13 - 2(0.999) - 0.948) = 1.005$$

$$z_2 = \frac{1}{10}(14 - 2(0.999) - 2(1.005)) = 0.999$$

So, after 2nd iteration,
(x,y,z)=(0.999, 1.005, 0.999)

Convergence

The values are almost stable,
Then solution is

x≈1, y≈1, z≈1

And the final answer

x=1, y=1, z=1