

**BCA [5TH SEM]**  
**COURSE CODE : BCA-504**

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# **NUMERICAL METHODS**

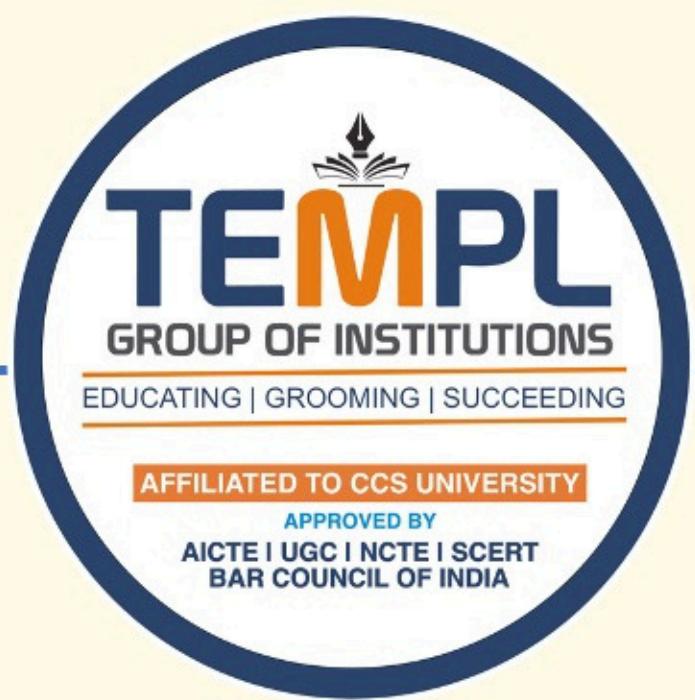
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# UNIT-4



## SOLUTION OF LINEAR EQUATION

# LINEAR EQUATION



**A linear equation in n unknowns is an equation of the form**

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$$

**where  $a_1, a_2, \dots, a_n$  and  $b$  are real (or complex) constants and  $x_1, x_2, \dots, x_n$  are unknowns.**

**Ex:**

$$2x + 3y = 5$$

**Linear equation in 2 variables**

$$x - y + z = 4$$

**Linear equation in 3 variables**

**A system of linear equations consists of several such equations taken together**

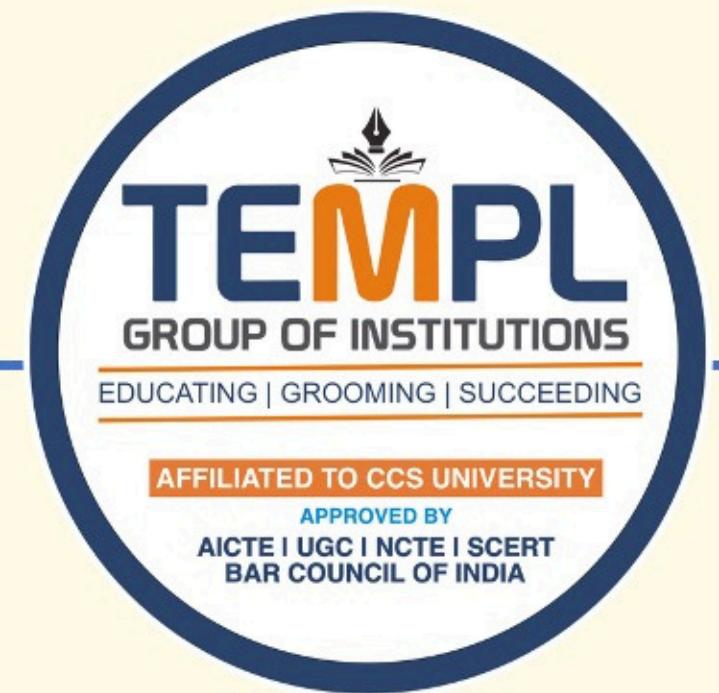
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

⋮

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

# MATRIX REPRESENTATION



In compact form,

$$AX = B$$

where

**A = coefficient matrix (m×n)**

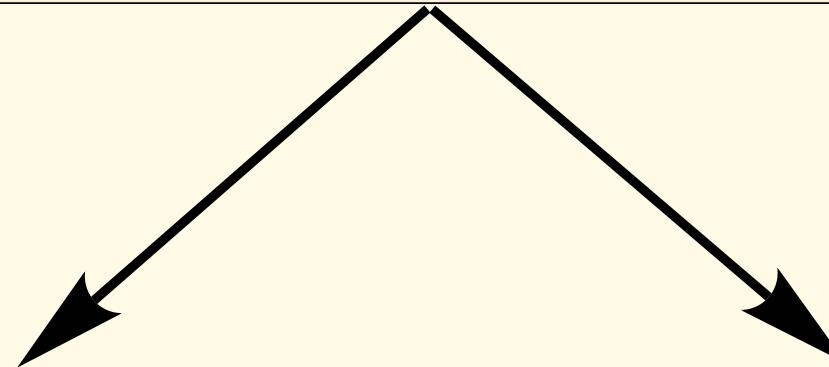
**$X = [x_1, x_2, \dots, x_n]^T$  = vector of unknowns**

**$B = [b_1, b_2, \dots, b_m]^T$  = vector of constants**

# SOLUTION OF LINEAR EQUATION



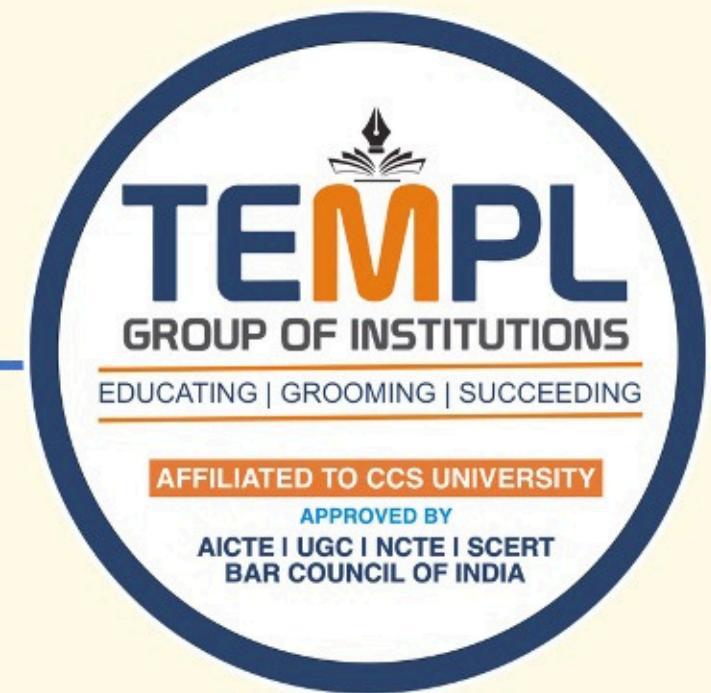
## SOLUTION OF LINEAR EQUATION



Gauss's Elimination Method

Gauss's Seidal Iterative Method

# GAUSS'S ELIMINATION METHOD

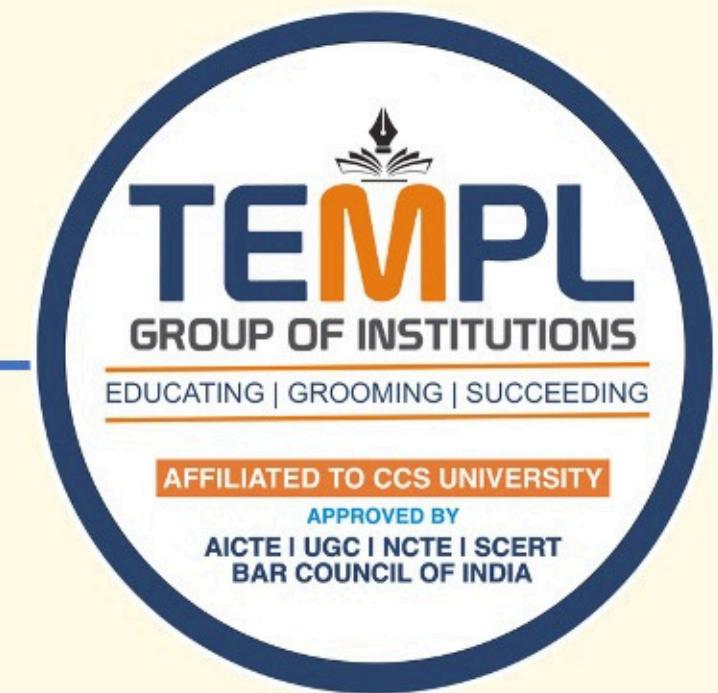


**Gauss's Elimination Method is a direct method of solving a system of linear equations.**

**It works by reducing the system to an upper triangular form using forward elimination, and then solving the equations using back substitution.**

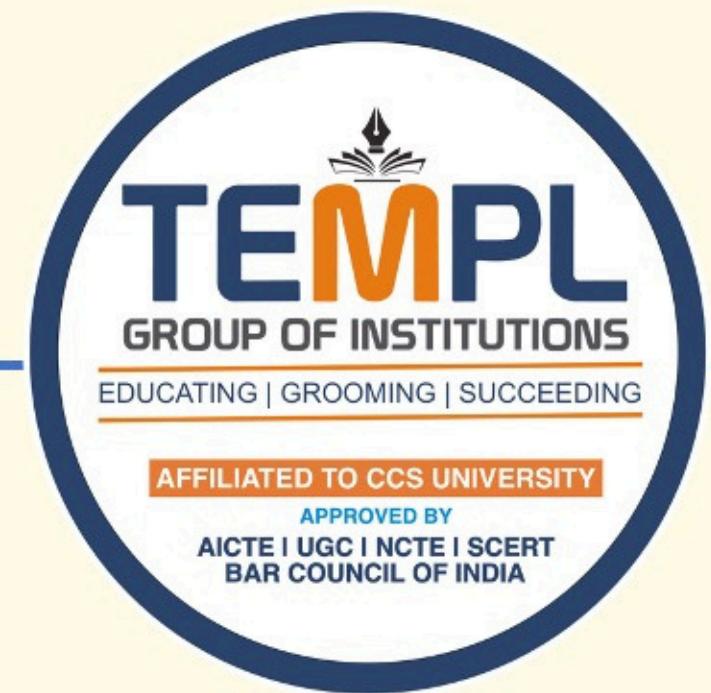
**Gauss elimination is named after German mathematician and scientist Carl Friedrich Gauss.**

# STEPS OF GAUSS'S ELIMINATION METHOD



- » **Form the Augmented Matrix [A | B].**
- » **Forward Elimination**
  - Choose a pivot (diagonal element).
  - Make all elements below pivot = 0 by row operations.
  - Repeat for each column until upper triangular form is obtained.
- » **Back Substitution**
  - Start from last equation (with one variable).
  - Substitute upward to find all variables.

# EXAMPLE OF GAUSS'S ELIMINATION METHOD



**The system of equations are**

$$2x + y - z = 8$$

$$-3x - y + 2z = -11$$

$$-2x + y + 2z = -3$$

**Then Augmented Matrix is**

$$\left[ \begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array} \right]$$

**Now, Forward Elimination**

Make pivot = 2 (first element)  
Eliminate below it

$$R2 \rightarrow R2 + \frac{3}{2}R1, \quad R3 \rightarrow R3 + R1$$

$$\left[ \begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 2 & 1 & 5 \end{array} \right]$$

**Next, pivot = 0.5 (row 2)**

**Eliminate below it**

$$R3 \rightarrow R3 - 4R2$$

$$\left[ \begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 0 & -1 & 1 \end{array} \right]$$

**Now we have upper triangular form**

**Now, Back Substitution**

**From last row,**  
 $-z=1 \Rightarrow z=-1$

**From second row**

$$0.5y+0.5z=1$$

$$0.5y-0.5=1 \Rightarrow y=3$$

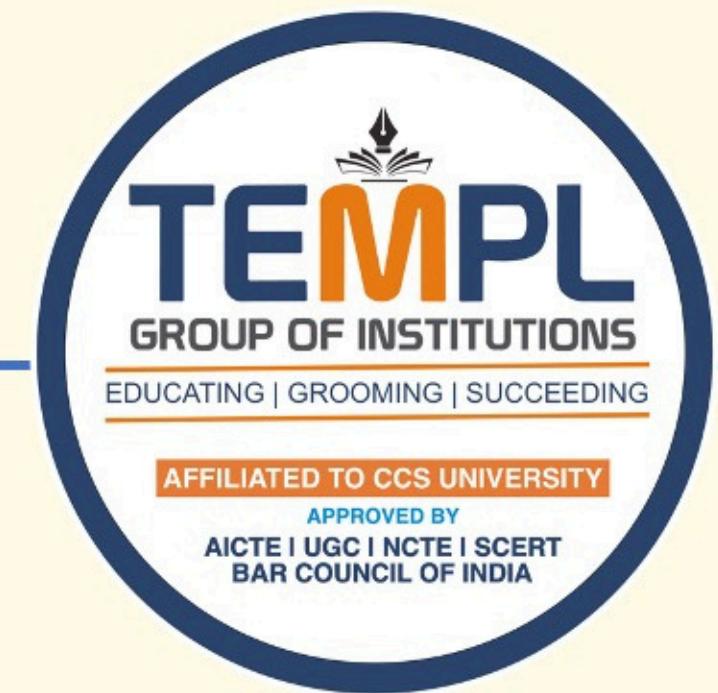
**From first row**  
 $2x+y-z=8$

$$2x+3-(-1)=8 \Rightarrow 2x+4=8 \Rightarrow x=2$$

**And the final answer**

$$x=2, y=3, z=-1$$

# GAUSS'S SEIDAL ITERATIVE METHOD

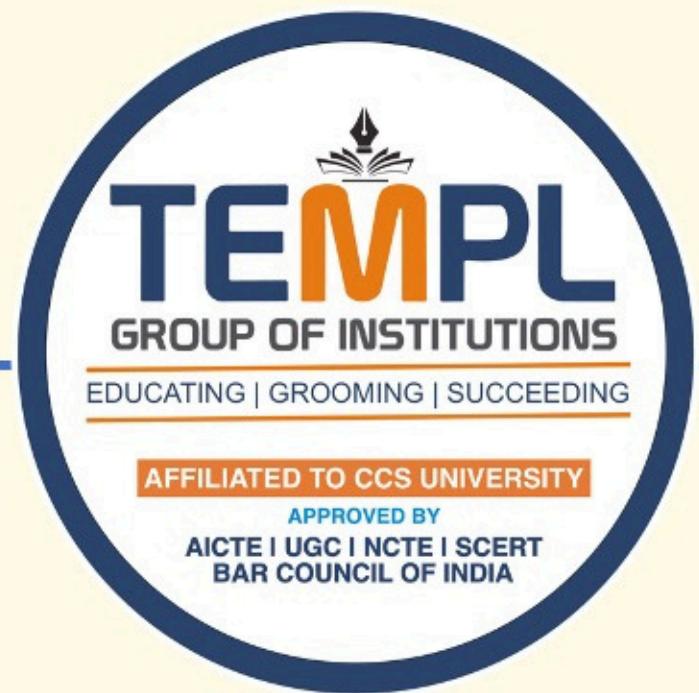


**Gauss Seidel Method is an iterative method used to solve a system of linear equations.**

**Instead of solving directly (like Gauss elimination), it starts with an initial guess and improves the solution step by step until the values converge (become stable).**

**Gauss Seidel is named after the German mathematicians Carl Friedrich Gauss and Philipp Ludwig von Seidel.**

# STEPS OF GAUSS'S SEIDAL ITERATIVE METHOD



- **Rearrange Equations**
  - Write each equation so that one variable is expressed in terms of the others.
- **Initial Guess**
  - Assume starting values for all variables (usually 0, 0, 0...).
- **Iterative Update**
  - Compute  $x_1$  using the latest values of other variables.
  - Compute  $x_2$  using the new  $x_1$ .
  - Compute  $x_3$  using the new  $x_1, x_2$ .
  - Continue for all variables.
- **Repeat Iterations**
  - Perform steps until the values stop changing (difference is very small).
- **Final Solution**
  - The stable (converged) values are taken as the solution of the system.

# EXAMPLE OF GAUSS'S SEIDAL ITERATIVE METHOD

The system of equations are

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$

Then Rearrange Equations

$$x = \frac{1}{10}(12 - y - z)$$

$$y = \frac{1}{10}(13 - 2x - z)$$

$$z = \frac{1}{10}(14 - 2x - 2y)$$

Now, Initial Guess

Take  $(x_0, y_0, z_0) = (0, 0, 0)$

## Iteration 1

$$x_1 = \frac{1}{10}(12 - 0 - 0) = 1.2$$

$$y_1 = \frac{1}{10}(13 - 2(1.2) - 0) = \frac{13 - 2.4}{10} = 1.06$$

$$z_1 = \frac{1}{10}(14 - 2(1.2) - 2(1.06)) = \frac{14 - 2.4 - 2.12}{10} = 0.948$$

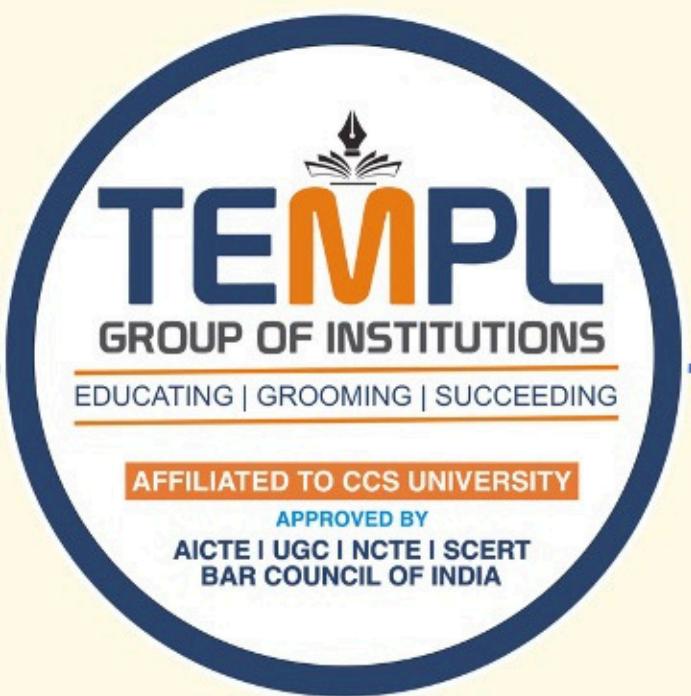
So, after 1st iteration,  
 $(x, y, z) = (1.2, 1.06, 0.948)$

## Iteration 2

$$x_2 = \frac{1}{10}(12 - 1.06 - 0.948) = 0.999$$

$$y_2 = \frac{1}{10}(13 - 2(0.999) - 0.948) = 1.005$$

$$z_2 = \frac{1}{10}(14 - 2(0.999) - 2(1.005)) = 0.999$$



So, after 2nd iteration,  
 $(x, y, z) = (0.999, 1.005, 0.999)$

## Convergence

The values are almost stable,  
Then solution is

$$x \approx 1, y \approx 1, z \approx 1$$

And the final answer

$$x = 1, y = 1, z = 1$$