BCA [5TH SEM]

COURSE CODE : BCA-504

NUMERICAL METHODS

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NUMERICAL DIFFERENTIATION AND INTEGRATION

NUMERICAL DIFFERENTIATION

Numerical differentiation is a numerical technique used to approximate the derivative of a function when the function is known only at discrete tabulated values and the analytical form of the function is not available or difficult to differentiate.

It is based on difference formulas derived from interpolation methods such as Newton's forward, backward difference formulas and many more.

DIRECT METHODS FOR DERIVATIVES

DIRECT METHODS FOR DERIVATIVES

- Derivative Formulae for Equal Intervals
- Derivative Formulae for Central Intervals
- Derivative Formulae for Unequal Intervals

DERIVATIVE FORMULAE FOR EQUAL INTERVALS

DERIVATIVE FORMULAE FOR EQUAL INTERVALS

- **Newton's Forward Difference Formula**
- **Newton's Backward Difference Formula**

NEWTON'S FORWARD DIFFERENCE FORMULA

Newton's Forward Difference Formula is an interpolation formula used to find the value of a function near the beginning of a given data table when the independent variable values are equally spaced.

It is based on forward differences and is suitable when the value of x is close to the first observation.

> The Newton's Forward Difference Formula was developed by both Sir Isaac Newton and Scottish mathematician James Gregory around the 17th century.

NEWTON'S FORWARD DIFFERENCE FORMULA

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Newton's Forward Difference Formula is

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

Where,

$$u = \frac{x - x_0}{h}$$

NEWTON'S BACKWARD DIFFERENCE FORMULA

Newton's Backward Difference Formula is used to find the value of a function near the end of the given data table when the values of the independent variable are equally spaced.

It is based on backward differences and is suitable when x is close to the last observation.

The Newton's Backward Difference Formula was developed by both Sir Isaac Newton and Scottish mathematician James Gregory around the 17^{th} century.

NEWTON'S BACKWARD DIFFERENCE FORMULA

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Newton's Backward Difference Formula is

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

Where,

$$u = \frac{x - x_n}{h}$$

DERIVATIVE FORMULAE FOR CENTRAL INTERVALS

DERIVATIVE FORMULAE FOR CENTRAL INTERVALS

Bessel's Formula

Stirling's Formula

BESSEL'S FORMULA

Bessel's Formula is an interpolation formula used to find the value of a function when the given value of x lies near the midpoint of two central values of the table.

It is particularly useful when the value of u is close to 0.5.

The Bessel's Formula was developed by German astronomer and mathematician Friedrich Wilhelm Bessel around the 17^{th} century.

BESSEL'S FORMULA

Bessel's Formula is

$$y(x) = \frac{y_0 + y_1}{2} + \left(u - \frac{1}{2}\right)\Delta y_0 + \frac{u(u-1)}{2!} \cdot \frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}$$

$$+ \frac{(u - \frac{1}{2})u(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

Where,

$$u = \frac{x - x_0}{h}$$

STIRLING'S FORMULA

Stirling's Formula is a central difference interpolation formula used when the value of x lies very close to the central value of the table.

It is derived by taking the average of Newton's forward and backward formulas.

The Bessel's Formula was developed by the Scottish mathematician James Stirling around the 17th century.

STIRLING'S FORMULA

Stirling's Formula is

$$y(x) = y_0 + u\frac{\Delta y_0 + \Delta y_{-1}}{2} + \frac{u^2}{2!}\Delta^2 y_{-1}$$

$$+\frac{u(u^2-1)}{3!}\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \cdots$$

Where,

$$u = \frac{x - x_0}{h}$$

DERIVATIVE FORMULAE FOR UNEQUAL INTERVALS

DERIVATIVE FORMULAE FOR UNEQUAL INTERVALS

- Newton's Divided Difference Formula
- Lagrange's Formula

NEWTON'S DIVIDED DIFFERENCE FORMULA

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Newton's Divided Difference Formula is an interpolation formula used to find the value of a function when the values of the independent variable are unequally spaced.

It is based on divided differences, and the formula can be extended easily when new data points are added.

> The Newton's Divided Difference Formula was developed by Sir Isaac Newton around the 17th century.

NEWTON'S DIVIDED DIFFERENCE FORMULA

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Newton's Divided Difference Formula is

$$y(x) = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2]$$

$$+(x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3] + \cdots$$

Where,

$$[x_i, x_j] = \frac{y_j - y_i}{x_j - x_i}$$

$$[x_i, x_j, x_k] = \frac{[x_j, x_k] - [x_i, x_j]}{x_k - x_i}$$

LAGRANGE'S FORMULA

Lagrange's Interpolation Formula is used to find an approximate value of a function when the values of the independent variable are unequally spaced.

It expresses the interpolating polynomial as a linear combination of basis polynomials.

The Newton's Divided Difference Formula was developed by the Italian French mathematician Joseph Louis Lagrange around the 18th century.

LAGRANGE'S FORMULA

Lagrange's Formula is

If three points are given:

$$(x_0, y_0),\ (x_1, y_1),\ (x_2, y_2)$$

Then,

$$y(x) = y_0 \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + y_1 \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + y_2 \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

NUMERICAL INTEGRATION

Numerical Integration is a numerical technique used to find the approximate value of a definite integral when

- the function is known only at discrete (tabulated) points, or
- the integral cannot be evaluated analytically.

In numerical integration, the area under the curve is approximated using algebraic formulas such as Trapezoidal rule and Simpson's rules.

METHODS FOR NUMERICAL INTEGRATION

METHODS FOR NUMERICAL INTEGRATION

- Trapezoidal rule
- Simpson $1/3^{rd}$ rule
- Simpson $3/8^{rd}$ rule

TRAPEZOIDAL RULE

The Trapezoidal Rule is a numerical integration method used to find the approximate value of a definite integral by dividing the area under the curve into a series of trapeziums (trapezoids).

There's no single "founder" for the Trapezoidal Rule, as it's an ancient concept, used by Babylonian astronomers over 2,000 years ago (before 50 BCE) and rediscovered/formalized by later mathematicians like Leonhard Euler and Colin Maclaurin around the 1740s, with its modern numerical analysis form evolving from Riemann Sums and methods for approximating integrals.

TRAPEZOIDAL RULE

Trapezoidal rule is

$$\int_a^b y\,dx \approx \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

Where,

$$h = \frac{b-a}{n}$$

SIMPSON 1/3RD RULE

Simpson's 1/3rd Rule is a numerical integration method that approximates the curve by parabolic arcs.

It gives more accurate results than the trapezoidal rule.

The Newton's Divided Difference Formula was developed by the English mathematician Thomas Simpson around the 18th century.

SIMPSON 1/3RD RULE

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Simpson 1/3rd rule is

$$\int_a^b y\,dx \approx \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$$

SIMPSON 3/8RD RULE

Simpson's 3/8rd Rule approximates the curve using cubic polynomials and is used when the number of sub intervals is a multiple of 3.

The Newton's Divided Difference Formula was developed by the English mathematician Thomas Simpson around the 18th century.

SIMPSON 3/8RD RULE

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Simpson 3/8rd rule is

$$\int_a^b y\,dx \approx \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + \cdots) + 2(y_3 + y_6 + \cdots)\right]$$